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CALCULUS.

172. Proposed by F. P. MATZ, Sc. D., Ph. D., Professor of Mathematics and Astronomy in Defiance College, Defiance, O.

$$\text{Solve } x \frac{dy}{dx} = \frac{y}{y^{-1} - \log x}.$$

Solution by W. W. BEMAN, A. M., Professor of Mathematics at the University of Michigan, Ann Arbor, Mich.

Writing the equation in the form $y \frac{dx}{x} + \log x \, dy = \frac{dy}{y}$, we get $y \log x = \log cy$,
or, $x^y = cy$.

Also solved by G. W. Droke, Fayetteville, Ark.; M. E. Graber, A. B., Instructor in Mathematics and Physics, Heidelberg University, Tiffin, O.; O. W. Anthony, DeWitt Clinton High School, New York City; G. W. Greenwood, B. A. (Oxon), Professor of Mathematics and Astronomy, McKendree College, Lebanon, Ill.; G. B. M. Zerr, A. M., Ph. D., Parsons, W. Va.

MECHANICS.

161. Proposed by W. J. GREENSTREET, A. M., Editor of The Mathematical Gazette, Stroud, England.

Four equal uniform smoothly jointed rods length a , and width w , form a rhombus $ABCD$, A and C being in contact with two vertical walls b feet apart. An elastic string, natural length x , modulus λ , keeps the figure in position. The angle of friction at A and C is $\tan^{-1}p$. When the rhombus is just about to slip, find the angle A , and the angle between AB and the vertical.

Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

Suppose the rhombus to be held in form by two strings AC , BD in a state of tension and that the rhombus is in a plane perpendicular to the walls. Let T , T' be the tensions in BD , AC ; then the virtual work $T' \cdot \delta AC + T \cdot \delta BD = 0$.

$$\text{But } AC^2 + BD^2 = 4a^2, \therefore AC \cdot \delta AC + BD \cdot \delta BD = 0.$$

$$\therefore T' \cdot BD = T \cdot AC \text{ or } T' = T \cdot AC / BD.$$

Let BD make an angle θ with the vertical. Then $b = AC \cos \theta$ or $AC = b \sec \theta$, $BD = x(1 + T/\lambda) = x_1$. Let R , S be the reactions at A , C . Revolving horizontally, $R + S = 2T' \cos \theta$. Revolving vertically, $(R + S)p = 4w$.

$$\therefore T' = \frac{2w}{p \cos \theta} = \frac{Tb \sec \theta}{x_1} \text{ or } T = \frac{2wx_1}{pb}.$$

$$\therefore x_1 = x \left(1 + \frac{2wx_1}{pb\lambda} \right) \text{ or } x_1 = \frac{pb\lambda x}{pb\lambda - 2wx}.$$

$$x_1 = 2a \sin \frac{1}{2}A \text{ or } A = 2 \sin^{-1}(x_1/2a). \quad AB \text{ makes with the vertical an angle } \frac{1}{2}B + \theta = \frac{1}{2}\pi - \frac{1}{2}A + \theta.$$

162. Proposed by B. F. FINKEL, A. M., M. Sc., Professor of Mathematics and Physics, Drury College, Springfield, Mo.

Show that the velocity, v , of a wave along the surface of a liquid whose

depth is not less than the wave length, λ , whose density is δ , and surface tension T , is $v^2 = \frac{g\lambda}{2\pi} + \frac{2\pi T}{\lambda\delta}$.

Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

Let ϕ denote the velocity. Taking the axis of x in the undisturbed surface and drawing the axis of y vertically downward we get, $C + p/\delta - d\phi/dt$. Since the motion is small, we can neglect the square of the velocity. The equation of continuity is $d^2\phi/dx^2 + d^2\phi/dy^2 = 0$.

$$\therefore \phi = \Sigma A_n e^{-my} \cos(nt - mx). \quad \therefore C + p/\delta = gy + \Sigma n A_n e^{-my} \sin(nt - mx).$$

At the free surface $C\delta + p$ is the excess of the pressure in the liquid over the atmospheric pressure due to the curvature and tension of the surface. Let y_0 = the vertical displacement of the free surface, then $d^2 y_0/dx^2$ is the curvature, approximately; hence at the free surface $T/\delta \cdot d^2 y_0/dx^2 = C + p/\delta = gy_0 + \Sigma n A_n \sin(nt - mx)$. Let $y = 0$, then $dy_0/dt = d\phi/dy = -\Sigma m A_n \cos(nt - mx)$.

$$\therefore y_0 = -\Sigma (m/n) A_n \sin(nt - mx). \quad \therefore d^2 y_0/dx^2 = \Sigma (m^3/n) A_n \sin(nt - mx).$$

Substituting in the equation at the free surface we must have $m^3 T/n\delta = -(m/n)g + n$ or $n^2/m^2 = g/m + mT/\delta$. Now $n + 2\pi v/\lambda$, $m = 2\pi/\lambda$.

$$\therefore v^2 = g\lambda/2\pi + 2\pi T/\lambda\delta.$$

Also solved by M. E. Graber, A. B., Instructor in Mathematics and Physics, Heidelberg University, Tiffin, O.

AVERAGE AND PROBABILITY.

63. Proposed by COL. CLARKE.*

Three points are taken at random one on each of the three faces of a tetrahedron. What is the chance that a plane passing through them cuts the fourth edge? [From Williamson's *Integral Calculus*, page 410.]

Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

Let $ABCD$ be the tetrahedron, P a random point in the face ADC , Q a random point in the face ADB . Through A , P draw AF meeting DC in F ; through A , Q draw AE meeting BD in E . Let $DB = a$, $DC = b$, $DE = x$, $DF = y$. On DB and DC take G and H on the same side of EF so that $EG = dx$, $FH = dy$.

The chance that a point falls in the triangle AEG is dx/a , that it falls in the triangle AFH is dy/b . In order that the plane through P , Q may cut AC and AB or the face ABC , the third point must fall on the triangle DEF . The chance of this is $(x/a)(y/b) = xy/ab$.

$\therefore$ The whole chance of the plane through P , Q cutting AB and AC is

$$p = \int_0^a \int_0^b \frac{xy}{ab} dx dy / \int_0^a \int_0^b dx dy = \frac{1}{a^2 b^2} \int_0^a \int_0^b xy dx dy = \frac{1}{4}.$$

Similarly the chance of AC and BC being cut is $\frac{1}{4}$, and the chance of AB and BC being cut is $\frac{1}{4}$. $\therefore$ The chance of ABC being cut $= \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4}$.

*Vol. V, 1898, p. 120.